Since the main features of the reflectance spectrum can all be assigned to AP, it is concluded that the TCP effect is of second order. This justifies the homogeneous model employed here as opposed to a composite model which might have been used to account for the TCP. It should also be noted that the values for k in the regions of high transmission are very difficult to determine accurately by the present method. This is because it has been assumed that the only resonances present are those observed in the reflectance spectrum and that these resonances have the assumed damped harmonic oscillator form. Deviation from these assumptions would introduce significant errors in the value of k in the very weak absorbing regions.

Conclusions

A dispersion equation curve-fitting technique in conjunction with normal spectral reflectance measurements was used to determine the optical constants of propellant-grade ammonium perchlorate. Although the stringent requirements for sample surface preparation were not satisfied completely the optical constants obtained in this manner represent the only, therefore, the best data available. Furthermore, the values of reflectance calculated from these optical constants are felt to be very representative of as-received propellant-grade AP. Radiative transfer in AP composite propellants is typically dominated by geometric optics scattering (which is dependent on reflectance data and not the precise optical constants). Therefore, the presented results will be very useful for modeling radiative transfer in aluminized composite (AP) propellants in an effort to reduce unwanted aluminum agglomeration caused by radiative heating.

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Identification of Structural Dynamic Systems with Nonproportional Damping

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Introduction

A LINEAR structural dynamic system is usually represented mathematically by a set of second-order linear differential equations. The inertia, damping, and stiffness effects are quantified in terms of mass, damping, and stiffness matrices whose order is equal to the number of degrees of freedom chosen in modeling the physical system. An identification problem involving such systems reduces to the determination of these matrices from a given set of information about the dynamic behavior of the system. For example, the information can be in the form of the system output in the time domain for a specified input. Although a substantial number of techniques have been developed by various researchers to identify undamped or proportionally damped systems, methods capable of handling generalized damping matrices are relatively scarce. Currently existing procedures in this area include the estimation of system matrices using time domain output,[1-3] frequency domain output,[4] and modal parameters.[5,6]

In this paper a different approach to the problem is presented. The technique is based on the assumption that either some or all of the elements of the mass matrix are known. It is also assumed that the complex eigenvalues and the complex eigenvectors of the system have been measured. The remaining elements of the mass matrix, the stiffness matrix, and the damping matrix are determined by minimizing the Euclidean norm of a matrix that assures the satisfaction of the equations of the eigenvalue problem and the appropriate orthogonality conditions. The minimization of the norm has been performed subject to the constraints of symmetry.

Eigenvalue Problem and Identification of System Matrices

The eigenvalue problem for a system with n-degrees-of-freedom can be stated as follows:

$$MU+CV+KW=0 \tag{1}$$

$$U=[\lambda_1^2\phi_1,\lambda_2^2\phi_2\ldots\lambda_n^2\phi_n] \tag{2}$$

$$V=[\lambda_1\phi_1,\lambda_2\phi_2\ldots\lambda_n\phi_n] \tag{3}$$

$$W=[\phi_1,\phi_2\ldots\phi_n] \tag{4}$$

In these equations λ_i and ϕ_i represent the ith eigenvalue and eigenvector, respectively. It is to be noted that in this equa-

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tion only n of the $2n$ modes of the corresponding first-order system are included; when Eq. (1) is satisfied for a given set of M, C, and K with the first n modes, it is also satisfied for their complex conjugates. The identification problem consists of finding the matrices M, C, and K when all the eigenvalues and eigenvectors are known through experiments or otherwise. If all the eigenvalues and eigenvectors are not known and/or all the eigenvector components are not known, the missing part of the data must be synthesized. If incorrect values of mass, damping, and stiffness matrices are substituted in Eq. (1), the right-hand side will not be zero but will become an error matrix E with

$$E_{ij} = \sum_{m=1}^{n} (M_{im}U_{mj} + C_{im}V_{mj} + K_{im}W_{mj}) \tag{5}$$

An identification procedure can now be formulated in which the Euclidean norm of E is minimized by varying the elements of the system matrices. Values of M, C, and K which yield the least error norm are accepted as optimum estimates. If the matrices thus identified are such that they satisfy the eigenvalue problem with respect to the measured modal parameters exactly ($E=0$), the orthogonality conditions are also satisfied exactly. If the error in the eigenvalue problem is not zero, orthogonality is satisfied approximately. Even in this case, however, orthogonality is satisfied exactly with respect to the identified modal parameters (computed from identified matrices), which in turn are approximately equal to the measured modal parameters. The orthogonality conditions are thus not a part of explicit constraints. The remaining constraint equations concern symmetry:

$$M - M^T = 0; \quad C - C^T = 0; \quad K - K^T = 0 \tag{6}$$

Then, the constrained objective function to be minimized becomes

$$\beta = \alpha^E + \alpha^M + \alpha^C + \alpha^K \tag{7}$$

$$\alpha^E = \sum_{j=1}^{n}\sum_{i=1}^{n} |E_{ij}|^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} E_{ij}\bar{E}_{ij}$$

$$\alpha^M = \sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij}^M (M_{ij} - M_{ji}) \tag{8}$$

$$\alpha^C = \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij}^C (C_{ij} - C_{ji}); \quad \alpha^K = \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij}^K (K_{ij} - K_{ji}) \tag{9}$$

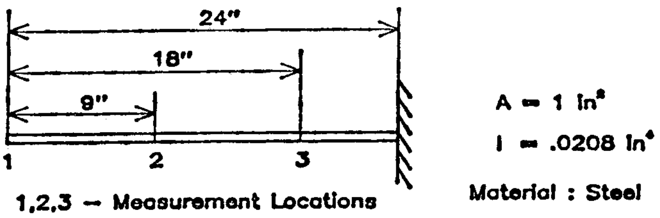

Fig. 1 Cantilever beam used in experiments.

Table 1 Identified matrix elements for the two-degree-of-freedom system

Case	M_{11}	M_{12}	M_{22}	C_{11}	C_{12}	C_{22}	K_{11}	K_{12}	K_{22}
No error in eigenparameters	100	0	200	40	−20	20	500	−400	400
−1% error in λ's +5% error in Φ's	100	0	200	39.6	−19.8	19.8	490	−392	392
+1% error in λ's +5% error in Φ's	100	0	200	40.4	−20.2	20.2	510	−408	408
Exact values	100	0	200	40	−20	20	500	−400	400

Table 2 System matrices

Model	Element index	11	21	22	31	32	33
A priori	Mass[a]	0.324	0.191	1.061	−0.095	0.149	0.942
	Damping[b]	—	—	—	—	—	—
	Stiffness[c]	0.238	−0.553	1.583	0.496	−2.098	5.567
Identified Case 1, all elements of M known	Mass[a]	0.324	0.191	1.061	−0.095	0.149	0.942
	Damping[b]	0.525	0.133	1.923	0.023	−0.229	2.002
	Stiffness[c]	0.301	−0.669	1.681	0.496	−1.604	2.942
Identified Case 2, only diagonal elements of M known	Mass[a]	0.324	0.121	1.061	0.082	−0.250	0.942
	Damping[b]	0.571	−0.086	2.243	0.438	−1.226	2.494
	Stiffness[c]	0.450	−1.117	2.939	1.212	−3.396	4.785

[a]N s^2/m, [b] N s/m, [c]$\times 10^6$ N/m.

Table 3 Comparison of modal parameters

Mode no.	Experimental values λ	Experimental values ϕ^T	Identified values (case 1) λ	Identified values (case 1) ϕ^T	Identified values (case 2) λ	Identified values (case 2) ϕ^T
1	$-0.73+j158.3$	[1 0.52 0.12]	$-0.73+j158.2$	[1 0.52 0.12]	$-0.73+j158.3$	[1 0.52 0.12]
2	$-0.76+j947.5$	[1 −0.55 −0.73]	$-0.76+j962.4$	[1 −0.46 −0.67]	$-0.76+j947.5$	[1 −0.55 −0.73]
3	$-1.62+j2693.6$	[1 −0.69 0.83]	$-1.66+j2762.4$	[1 −0.63 0.69]	$-1.62+j2694.0$	[1 −0.69 0.83]

In these equations Lagrange multipliers are defined as follows:

$$\gamma^M = -(\gamma^M)^T, \quad \gamma^C = -(\gamma^C)^T, \quad \gamma^K = -(\gamma^K)^T \tag{10}$$

Minimization and simplification results in a set of homogeneous linear equations

$$ML_1 + L_1M + CL_2 + L_2^TC + KL_3 + L_3^TK = 0$$

$$ML_2^T + L_2M + CL_4 + L_4C + KL_5 + L_5^TK = 0$$

$$ML_3^T + L_3M + CL_5^T + L_5C + KL_6 + L_6K = 0 \tag{11}$$

where

$$L_1 = Re(UU^*), \quad L_2 = Re(VU^*), \quad L_3 = Re(WU^*)$$

$$L_4 = Re(VV^*), \quad L_5 = Re(WV^*), \quad L_6 = Re(WW^*) \tag{12}$$

and an asterisk denotes complex congugate transpose.

To obtain a nontrivial solution, however, at least one nonzero element of any one of the matrices M, C, and K must be specified. If this condition is satisfied, the set of equations in Eq. (11) can be solved by neglecting the rows obtained by differentiating β with respect to the known elements and substituting the values of these elements into the remaining equations, thus converting the system into an inhomogeneous one. If the total mass of the system is known and if the mass matrix is assumed to be diagonal, the preceding procedure can be used to get a unique solution. This is accomplished by first assigning an arbitrary value to one of the elements of the diagonal mass matrix and solving for the remaining unknowns in M, C, and K. The identified matrices are then multiplied by a scalar so that the sum of the diagonal elements of the revised mass matrix is equal to the total mass and the ratios between the diagonal elements are preserved.

Numerical Results

As a first step, the performance of the described identification method is evaluated by assuming a set of M, C, K and determining the eigenvalues and eigenvectors which in turn are treated as experimental data in the identification procedure. A comparison of the identified and assumed M, C, K (Table 1) for a simple case involving nonproportional damping illustrates satisfactory performance. Damping values that are not present in most a priori models are estimated in this procedure. As a second example, a cantilever beam (Fig. 1) is considered. Experimentally, eigenvalues and eigenvectors have been determined at three locations. A corresponding three DOF a priori finite element model has been constructed from a larger DOF finite element model and Guyan reduction method. Identification procedure has then been applied by first assuming that the entire mass matrix is known and equal to the a priori mass matrix (case 1). The generated experimental data constitute the input to the procedure. Later, the identification procedure has been repeated by using only the diagonal elements of the a priori matrix as known (case 2). A comparison of the results is illustrated in Tables 2 and 3. A simulation to a high degree of accuracy in case 2 is attributed to the increased freedom available in adjusting the system matrices especially where experimental data are available at relatively fewer degrees of freedom.

Cases in which experimental data are not available for all the modes were investigated by considering the test results for higher modes as unknown. A priori analytical modal parameters were substituted for the higher modes for which experimental data were assumed to be lacking. As before, the model identified in each case using a priori diagonal mass elements was found to reproduce the specified modal parameters very accurately.

In some cases where some of the coefficients of the stiffness matrix were assumed to be known a priori and the coefficients of the mass and damping matrices were identified, the results were not always satisfactory. Even though the matrices obtained satisfied the equations of the eigenvalue problem, they were found not to be positive-definite, which would be considered unacceptable for most physical systems.

Conclusions

An identification procedure has been develped to estimate the mass, damping, and stiffness matrices from experimental eigenvalues and eigenvectors. The method is based on the minimization of the eigenvalue equation error subject to the conditions of symmetry of the matrices involved. If one or more nonzero coefficients in any of the matrices are given, the remaining coefficients in all the matrices can be found. The identified matrices are symmetric and satisfy the eigenvalue problem as accurately as possible. The method is applicable in all cases of damping, either proportional or nonproportional. Example problems are solved to indicate the validity of the approach. Results obtained in these examples also demonstrate that the procedure is capable of identifying systems that simulate experimental modal data to high levels of accuracy (when diagonal elements of the mass matrix were assumed to be known a priori). Application of the technique to systems with large degrees of freedom is still to be evaluated.

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Postbuckling of Thick Circular Plates with Edges Restrained Against Rotation

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Introduction

THE continuum finite-element formulations for the postbuckling analysis of circular plates is given in Ref. 1. A direct and simple finite-element formulation to study the

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